FEDERAL GOVERNMENT BONDS

Methodology for Calculating Federal Government Bonds Offered in Primary Auctions

The aim of this guide is to facilitate investor's understanding on some peculiarities concerning Brazilian domestic government bonds. We show below how interest and principal of federal government bond offered in primary auctions are calculated. Throughout the text there are tables with the main bond characteristics as well as the rules for rounding and truncation, in order to allow the accurate calculation of price, yield and quotation. In addition, for each bond we shall give a hypothetical example showing all the steps and details which must be followed.

Table 1 below shows the general characteristics of federal government bonds offered in the primary auctions.

TABLE 1
General Characteristics of Federal Government Bonds

Securities	Description	Indices	Coupon	Maximum Term	Type of Interest	Face Value on Maturity
LTN (National Treasury Bills)	Short-term, zero-coupon fixed rate bills	-	Zero coupon	About 24 months	Fixed	R$ 1,000
NTN-F (National Treasury Notes – Series F)	Long-term fixed rate coupon bonds	-	10% p.a. paid semi-annually	About 10 years		
NTN-B (National Treasury Notes – Series B)	Inflation-linked coupon bonds	IPCA Price Index Source: IBGE (www.ibge.gov.br)	6% p.a. paid semi-annually	About 40 years	Inflation linked	-
NTN-C (National Treasury Notes - Series C)	Inflation-linked coupon bonds	IGP-M Price Index Source: FGV (www.fgv.br)	6 or 12% p.a. paid semi-annually	About 25 years		-
LFT (Financial Treasury Bills)	Floating rate bills	Selic Interest Rate Source: Central Bank (www.bcb.gov.br)	Zero coupon	About. 5 years	Floating	-

Source: National Treasury.
Note: Type of securities => book entry, nominative and negotiable.

Besides the general characteristics of the securities, it is important to mention the formulae used in the calculation of price, yield and quotations. Since some of the securities in question pay semi-annual interest over the life of the bond and others do not, some particularities in pricing should be noted. Also, it is important to mention that in the case of floating rate securities, the quotation must be calculated before calculating the bond price itself.

Table 2 below shows the main formulae for calculating the security's quotation and price.

TABLE 2
Formulae Used for Calculating Federal Government Bonds Prices and Quotations

Securities	Price	Up to Date Nominal Value(UNV))	Quotation/Present Value
LTN (National Treasury Bills)	$\frac{1{,}000.00}{(1+ytm)^{\frac{bd}{252}}}$	-	-
NTN-F (National Treasury Notes - Series F)	$\left[\frac{1000\cdot(1.10^{0.5}-1)}{(1+ytm)^{bd1/252}}\right]+\left[\frac{1000\cdot(1.10^{0.5}-1)}{(1+ytm)^{bd2/252}}\right]+\ldots+\left[\frac{1000\cdot(1.10^{0.5}-1)}{(1+ytm)^{bdn/252}}\right]+\left[\frac{1000}{(1+ytm)^{bdn/252}}\right]$	-	
NTN-B (National Treasury Notes - Series B)	Quotation/100 * UNV Quotation = Present Value of the discounted cash flow	Accumulated Index of the IPCA from the reference date of the security (07/15/00) to settlement date	$\left[\frac{100\cdot(1.06^{0.5}-1)}{(1+ytm)^{bd1/252}}\right]+\left[\frac{100\cdot(1.06^{0.5}-1)}{(1+ytm)^{bd2/252}}\right]+\ldots+\left[\frac{100\cdot(1.06^{0.5}-1)}{(1+ytm)^{bdn/252}}\right]+\left[\frac{100}{(1+ytm)^{bdn/252}}\right]$
NTN-C (National Treasury Notes - Series C)	Quotation/100 * UNV Quotation = Present Value of the discounted cash flow	Accumulated Index of the IGP-M from the reference date of the security (07/01/00) to settlement date	
LFT (Financial Treasury Bills)	Quotation/100 * UNV Quotation = Present Value of the discounted principal	Accumulated Index of the SELIC rate from the reference date of the security (07/01/00) to settlement date	$\frac{100}{(1+ytm)^{\frac{bd}{252}}}$

Source: National Treasury.
The term *ytm* corresponds to the yield to maturity under the BD/252 % p.a. convention,
Note: In the case of a NTN-C with maturity on January 1st, 2031, the terms $(1.06)^{0.5}$ must be substituted by $(1.12)^{0.5}$.

Investors must follow some rules on truncation and rounding which will be fundamental for avoiding small price differences and allow the accurate calculation of price, yield and quotation. For illustration purposes, a hypothetical example for each security will be given highlighting all details and the attention needed to apply the appropriate pricing methodology.

National Treasury Bills - LTN

- *Price Calculation*

$$\text{Price} = \frac{1{,}000}{\underbrace{\left(1+\underbrace{\text{ytm}}_{\text{T}-4}\right)^{\underbrace{\left(\frac{\text{bd}}{252}\right)}_{\text{T}-14}}}_{\text{T}-6}}$$

where:

ytm = yield to maturity (BD/252 % p.a. convention => truncate to the 4th decimal place);
bd = number of business days between settlement date (inclusive) and maturity date (exclusive).

Example:

Maturity Date: 07/01/2010

Purchase Date: 05/20/2008

Settlement Date: 05/21/2008

Yield to Maturity: 14.3600% p.a.

Business Days between 05/21/2008 and 07/01/2010: 532

$$\text{Price} = \frac{1{,}000}{\left(1+\underbrace{14.3600\%}_{T-4}\right)^{\underbrace{\left(\frac{532}{252}\right)}_{T-14}}} = \underbrace{753.315323}_{\textit{truncate to the 6th decimal place}}$$

- *Yield to Maturity Calculation*

Conversely, the yield can be obtained on the basis of the price:

$$Yield = \left[\frac{1{,}000}{753.315323}\right]^{\frac{252}{532}} - 1 = \underbrace{14.3600}_{T-4}\%\ p.a.$$

Financial Treasury Bills – LFT

- *Quotation Calculation*

$$Quotation = \frac{100}{\left(1 + \underbrace{discount\ rate}_{T-4}\right)^{\underbrace{\left(\frac{bd}{252}\right)}_{T-14}}} \ ; \ truncate\ to\ the\ 4th\ decimal\ place$$

where:

discount rate: BD/252 % p.a. convention => truncate to the 4th decimal place;
bd = number of business days between settlement date (inclusive) and maturity date (exclusive).

- *Price Calculation*

$$\text{Price} = \underbrace{Quotation\ (\%)}_{T-4} * \overbrace{UNV}^{T-6} \ ; \ truncate\ to\ the\ 6th\ decimal\ place$$

where:

UNV = up to date nominal value up to settlement date. The UNV is published daily in Central Bank's website (http://www.bcb.gov.br).

Example:

Maturity Date: 03/07/2014

Purchase Date: 05/20/2008

Settlement Date: 05/21/2008

Nominal Value on reference date (07/01/2000): R$ 1,000

Discount Rate: -0.0200% (BD/252 % p.a. convention => truncate to 4th decimal place)

Number of business days between 05/21/2008 and 03/07/2014: 1459

$$Quotation\ (\%) = \frac{100}{\left(1+\underbrace{(-0.0200\%)}_{T-4}\right)^{\left(\frac{1459}{252}\right)}} = \underbrace{100.1158}_{T-4}\ \%$$

UNV on 05/21/2008 = R$ 1,000 x Accumulated Selic Index between 07/01/2000 and 05/21/2008

$$= R\$1{,}000 * \underbrace{(3.4512018246800000)}_{R-16} = R\$\ \underbrace{3{,}451.201824}_{T-6}$$

$$= R\$\ \underbrace{3{,}451.201824}_{\text{T-6}}$$

$$\text{Price} = R\$\overbrace{\underbrace{3{,}451.201824}_{\text{T-6}}}^{\text{UNV on 05/21/2008}} \text{ x } \overbrace{\underbrace{(100.1158\%)}_{\text{T-4}}}^{\text{Quotation}} = R\$\ \underbrace{3{,}455.198315}_{\text{T-6}}$$

<u>National Treasury Notes – NTN-B</u>

- *Quotation Calculation*

$$\text{Quotation}(\%) = \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd1}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd2}{252}\right)}_{T-14}}}\right]}_{R-10} + \ldots + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bdn}{252}\right)}_{T-14}}}\right]}_{R-10}$$

Note: the numerator of each term should be **rounded** to six decimal places and the final result for each term, to ten decimal places.

where:

ytm = yield to maturity (BD/252 % p.a. convention => truncate to the 4th decimal place);
bd = number of business days between settlement date (inclusive) and maturity date (exclusive).

- *Price Calculation*

$$\text{Price} = \underbrace{Quotation\ (\%)}_{T-4} * \overbrace{projected\ UNV}^{T-6};\ truncate\ to\ the\ 6th\ decimal\ place$$

where:

Projected UNV = up to date nominal value (inflation index – accumulated IPCA from the reference date of 07/15/00 to the settlement date) projected to the settlement date.

Example:

Maturity Date: 08/15/2010

Purchase Date: 05/20/2008

Settlement Date: 05/21/2008

Reference date value (07/15/2000): R$ 1,000.00

Yield to Maturity: 8.2900% (BD/252 % p.a. convention => truncate to the 4th decimal place);

Projection for IPCA April/2008: 0.46%

UNV on 05/15/2008 = R$ 1,000 x index of accumulated IPCA between 07/15/2000 and 15th day of the current month

$$= R\$1{,}000 * \underbrace{(1.72692645947653)}_{R-16}$$

$$= \underbrace{\text{R\$}1{,}726.926459}_{T-6}$$

UNV on 05/21/2008 = UNV on 05/15/2008 x $(1 + \text{IPCA}_{\text{projection}})^{pr1}$

where:

$$pr1 = \frac{\textit{number of consecutive days (settlement date, 15th day of previous month)}}{\textit{number of consecutive days (15th day of the current month, 15th day of the following month)}}$$

$$pr1 = \frac{(05/21/2008 - 05/15/2008)}{(06/15/2008 - 05/15/2008)} = \frac{6}{31}$$

$$(\text{UNV on } 05/21/2008 = \text{R\$}\underbrace{1{,}726.926459}_{T-6} * \left(1 + \underbrace{0.46\%}_{R-2}\right)^{\underbrace{\left(\frac{6}{31}\right)}_{T-14}} = \text{R\$}\underbrace{1{,}728.461136}_{T-6}$$

Pay Date	Cash Flow	Present Value	Business Days
	R-6	R-10	
08/15/2008	2.956301	2.8998535976	61
02/15/2009	2.956301	2.7840057610	190
08/15/2009	2.956301	2.6770128972	314
02/15/2010	2.956301	2.5733184988	439
08/15/2010	102.956301	86.1471473965	564
	Quotation %(T-4) =>	97.0813	

$$\text{Quotation}(\%) = \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\underbrace{\left(\frac{61}{252}\right)}_{T-14}} \Big/ \left(1+\underbrace{8.2900\%}_{T-4}\right)\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\underbrace{\left(\frac{190}{252}\right)}_{T-14}} \Big/ \left(1+\underbrace{8.2900\%}_{T-4}\right)\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\underbrace{\left(\frac{314}{252}\right)}_{T-14}} \Big/ \left(1+\underbrace{8.2900\%}_{T-4}\right)\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\underbrace{\left(\frac{439}{252}\right)}_{T-14}} \Big/ \left(1+\underbrace{8.2900\%}_{T-4}\right)\right]}_{R-10} + \ldots$$

$$+ \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}\right)}^{\mathrm{R}-6}}{\underbrace{\left(\frac{564}{252}\right)}_{\mathrm{T}-14}} \Big/ \left(1+\underbrace{8.2900\%}_{\mathrm{T}-4}\right)\right]}_{\mathrm{R}-10} = \underbrace{97.0813}_{\mathrm{T}-4}$$

Note: the numerator of each term should be **rounded** to six decimal places and the final result for each term, to ten decimal places.

$$\text{Price} = \text{R\$}\,\underbrace{1{,}728.461136}_{T\text{-}6} * \overbrace{97.0813\%}^{T-4} = \text{R\$}\,\underbrace{1{,}678.012540}_{T-6}$$

- Coupon Payment Calculation

$$Interest\ coupon = UNV * interest\ rate\ factor$$

where:

interest rate factor: semi-annual effective coupon rate

Example:

Maturity Date: 05/15/2045

Coupon Payment Date: 05/15/2008

Nominal value on reference date (07/15/2000): R$ 1,000

Up to Date Nominal Value on 05/15/2008: R$ 1,726.926439

$$\text{Interest} = \text{R\$}\ \overbrace{1{,}726.926459}^{T-6} * \underbrace{0.02956301}_{\left(1.06^{0.5}-1\right)=>\text{round to the 8th decimal place}} = \text{R\$}\ \overbrace{51.053144}^{T-6}$$

<u>National Treasury Notes – NTN-C</u>

- *Quotation Calculation*[1]

$$\text{Quotation}(\%) = \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd1}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd2}{252}\right)}_{T-14}}}\right]}_{R-10} + \ldots + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}\right)}^{R-6}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bdn}{252}\right)}_{T-14}}}\right]}_{R-10}$$

Note: the numerator of each term should be **rounded** to six decimal places and the final result for each term, to ten decimal places.

[1] In the case of the NTN-C maturing 01/01/2031, the terms $(1.06)^{0.5}$ should be replaced by $(1.12)^{0.5}$.

where:

ytm = yield to maturity (BD/252 % p.a. convention => truncate to the 4th decimal place);
bd = number of business days between settlement date (inclusive) and maturity date (exclusive).

- *Price Calculation*

$$Price = \underbrace{Quotation\ (\%)}_{T-4} * \overbrace{projected\ UNV}^{\text{T-6}}\ ;\ truncate\ to\ the\ 6th\ decimal\ place$$

where:

Projected UNV = up to date nominal value (index of the accumulated IGP-M from the reference date of 07/01/00 to the settlement date) estimated to settlement date.

Example:

Maturity Date: 03/01/2011

Purchase Date: 05/20/2008

Settlement Date: 05/21/2008

Nominal value on the reference date (07/01/2000): R$ 1,000

Yield to Maturity: 6.9000% (BD/252 % p.a. convention => truncate to the 4th decimal place);

Projection of IGP-M May 2008: 1.75 %

UNV on 05/21/2008 = R$ 1.000 x index of the accumulated IGP-M since the reference date (07/01/2000) up to the 1st day of the current month

$$= R\$\,1{,}000 * \underbrace{(2.10280551851751)}_{T-16}$$

$$= \text{R\$}\,\underbrace{2{,}102.805518}_{T-6}$$

UNV on 05/21/2008 = UNV on 05/01/2008 x (1 + IGP-M $_{\text{projection}}$)^{pr1}

where:

$$pr1 = \frac{number\ of\ consecutive\ days\ (1st\ \ day\ of\ current\ month, settlement\ date)}{number\ of\ consecutive\ days\ (1st\ \ day\ of\ the\ current\ month,\ 1st\ \ day\ of\ the\ following\ month)}$$

$$pr1 = \frac{(05/21/2008 - 05/01/2008)}{(06/01/2008 - 05/01/2008)} = \frac{20}{31}$$

$$\text{UNV on } 05/21/2008 = R\$\ \underbrace{2{,}102.805518}_{T-6} * \left(1 + \underbrace{1.75\,\%}_{R-2}\right)^{\underbrace{\left(\frac{20}{31}\right)}_{T-14}} = R\$\ \underbrace{2{,}126.473734}_{T-6}$$

Pay Date	Cash Flow	Present Value	Business Days
	R-6	R-10	
01/09/2008	2.956301	2.9004761983	72
03/01/2009	2.956301	2.8053073742	198
09/01/2009	2.956301	2.7125428649	325
03/01/2010	2.956301	2.6263204830	447
09/01/2010	2.956301	2.5381301937	576
03/01/2011	102.956301	85.5153966416	701
	Quotation %(T-4) =>	99.0981	

$$\text{Quotation}(\%) = \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{72}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{198}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{325}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{447}{252}\right)}_{T-14}}}\right]}_{R-10} + \ldots$$

$$+ \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}-1\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{576}{252}\right)}_{T-14}}}\right]}_{R-10} + \underbrace{\left[\frac{\overbrace{100*\left(1.06^{0.5}\right)}^{R-6}}{\left(1+\underbrace{6.9000\%}_{T-4}\right)^{\underbrace{\left(\frac{701}{252}\right)}_{T-14}}}\right]}_{R-10} = \underbrace{99.0981}_{T-4}$$

Note: the numerator of each term should be **rounded** to six decimal places and the final result for each term, to ten decimal places.

$$\text{Price} = \text{R\$}\ \underbrace{2{,}126.473734}_{T-6} * \underbrace{99.0981\%}_{T-4} = \text{R\$}\underbrace{2{,}107.295067}_{T-6}$$

- Coupon Payment Calculation

$$\textit{Coupon payment} = \textit{UNV} * \textit{interest rate factor}$$

where:

interest rate factor: semiannual effective coupon rate

Example:

Maturity Date: 04/01/2021

Coupon Payment Date: 05/01/2008

Nominal Value on reference date (07/01/2000): R$ 1,000

Up to Date Nominal Value on 04/01/2008: R$ 2,088.388799

$$\text{Interest} = \text{R\$}\ \overbrace{2.088{,}388799}^{T-6} * \underbrace{0.02956301}_{(1.06^{0.5}-1)\text{=>}\textit{round to the 8th decimal place}} = \text{R\$}\ \overbrace{61.739058}^{T-6}$$

Note: In the case of a NTN-C 01/01/2031, the terms $(1.06)^{0.5}$ should be replaced by $(1.12)^{0.5}$. In the above example, the coupon would be worth R$ 121.754152 .

<u>National Treasury Notes – NTN-F</u>

- *Price Calculation*

$$\text{Price} = \underbrace{\left[\frac{\overbrace{1000\times\left(1.1^{0.5}-1\right)}^{R-5}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd1}{252}\right)}_{T-14}}}\right]}_{R-9} + \underbrace{\left[\frac{\overbrace{1000\times\left(1.1^{0.5}-1\right)}^{R-5}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bd2}{252}\right)}_{T-14}}}\right]}_{R-9} + \ldots + \underbrace{\left[\frac{\overbrace{1000\times\left(1.1^{0.5}\right)}^{R-5}}{\left(1+\underbrace{ytm}_{T-4}\right)^{\underbrace{\left(\frac{bdn}{252}\right)}_{T-14}}}\right]}_{R-9}$$

Note the numerator of each term should be **rounded** to five decimal places.

Example:

Maturity Date: 01/01/2014

Purchase Date: 05/20/2008
Settlement Date: 05/21/2008

Yield to maturity: 13.6600% (BD/252 % p.a. convention => truncate to the 4th decimal place);

Pay Date	Cash Flow	Present Value	Business Days
	R-6	R-9	
07/01/2008	48.80885	48.119371611	28
01/01/2009	48.80885	45.020757190	159
07/01/2009	48.80885	42.314735474	281
01/01/2010	48.80885	39.650299657	409
07/01/2010	48.80885	37.248144536	532
01/01/2011	48.80885	34.902737214	660
07/01/2011	48.80885	32.771550709	784
01/01/2012	48.80885	30.723628208	911
07/01/2012	48.80885	28.832967367	1036
01/01/2013	48.80885	27.044908383	1162
07/01/2013	48.80885	25.406432363	1285
01/01/2014	1048.80885	511.040083815	1415
	Unit Price (T-6) =>	903.075616	

$$Price = \text{R\$}\ 903.075616$$

- *Coupon Calculation*

$$\text{Interest} = \text{R\$}\ 1{,}000 * \underbrace{\left(1.10^{0.5} - 1\right)}_{R-8} = \text{R\$}\ \underbrace{48.808850}_{T-6}$$

Finally, Table 3 below summarizes the conventions of truncation and rounding which are needed to ensure the precise calculation of price, yield and quotation.

TABLE 3
Conventions for the Accurate Calculation of Price, Quotation and Rate of Federal Government Bonds

			Base-100			Base-1000
Criteria	**Variable**	**LTN**	**LFT**	**NTN-B**	**NTN-C**	**NTN-F**
R	Interest payment (semester)	-	-	6	6	5
R	Inflation forecast	-	-	2	2	-
R	Acummulated Selic rate	-	16	-	10	-
R	Flow of discounted payments	-	-	10	14	9
T	Accrual fraction	-	-	14	14	-
T	Acummulated price index	-	-	16	16	-
T	YTM (% p.a.)	4	4	4	4	4
T	UNV	-	6	6	6	-
T	Price	6	6	6	6	6
T	Day exponential (bd/252)	14	14	14	14	14
T	Quotation (%)	-	4	4	4	-
T	Financial value	2	2	2	2	2

Key: R - round; T - truncate
Source: National Treasury